

# The Gerasimov-Drell-Hearn Sum Rule and Current Algebras

Lay Nam Chang<sup>a1</sup>   Yigao Liang<sup>b2</sup>   Ron L. Workman<sup>a</sup>

<sup>a</sup> Department of Physics and Institute for High Energy Physics

Virginia Polytechnic Institute and State University

Blacksburg, Virginia 24061-0435

<sup>b</sup> Department of Physics and Astronomy

University of Rochester

Rochester, New York 14627

## Abstract

The status of the Gerasimov-Drell-Hearn sum rules for polarized inclusive photo-production on nucleons is reviewed. It is shown that results from currently available data compare favorably with an estimate based on an extended current algebra. Implications for integrals of spin-dependent structure functions are also briefly discussed.

---

<sup>1</sup>Presently at Physics Division, National Science Foundation, Arlington, Virginia 22230, under a contract with National BioSystems, Rockville, Maryland 20852.

<sup>2</sup>Present address: Integrated Decisions and Systems, 8500 Normandale Lake Boulevard, Suite 1840, Bloomington, MN 55437

The Gerasimov-Drell-Hearn (GDH) sum rule[1] has recently regained the attention of both theorists and experimentalists. This renewed interest is partly due to the construction of CEBAF. There now exist proposals for measurements which will check both this sum rule and its generalization to non-zero values of  $Q^2$ . On the theoretical side there are connections[2] to the Bjorken sum rule and the EMC measurements[3]. As discrepancies exist in the predicted low- $Q^2$  behavior, it will be useful to better understand the physics at  $Q^2 = 0$ .

The original GDH sum rule, which actually results from a superconvergence relation for the spin-flip amplitude in Compton scattering, states that

$$\frac{2\pi^2\alpha}{M^2}(\kappa_{p,n})^2 = \int_{\omega_0}^{\infty} \frac{[\sigma_{3/2}(\omega) - \sigma_{1/2}(\omega)]_{p,n}}{\omega} d\omega, \quad (1)$$

where,  $\kappa_p(\kappa_n)$  is the proton(neutron) anomalous magnetic moment,  $\omega$  is the laboratory photon energy,  $M$  is the nucleon mass, and  $\alpha$  is the fine structure constant. The left-hand-side represents the single nucleon contribution to the spin-flip amplitude, while the right-hand-side involves an integration over the difference of helicity 3/2 and helicity 1/2  $\gamma p$  total cross sections.

Karliner[4] examined Eq.(1) decomposed into isoscalar and isovector components. The isovector sum rule was found to be well-satisfied. The isoscalar component, which is predicted to be small, was found to be small but difficult to determine precisely. The isovector-isoscalar sum rule provided the most unexpected result. Karliner's

estimate of the integral of cross sections was positive; the value from the anomalous magnetic moments is negative. These conclusions have been reproduced in a more recent study[5].

The problem with the isovector-isoscalar sum rule was also noticed by Fox and Freedman in their extensive study of Compton scattering sum rules[6]. Earlier, Abarbanel and Goldberger[7] had noted that the GDH sum rule would be modified if there were a fixed pole at  $J = 1$  in the angular momentum plane. Fox and Freedman suggested that this discrepancy might be evidence for such a pole.

Such fixed poles can be attributed generally to non-trivial terms in the associated current commutators[8, 9]. In this context, the superconvergence relation giving rise to the GDH sum rule implies that electric charge densities commute with each other. Within the standard model, electric charge densities are bilinear in fermion fields, and a naive application of canonical anti-commutation relations substantiates this conclusion.

However, a more careful study carried out recently shows that the commutator actually does have additional terms[10]. Indeed, these terms have quantum numbers that can give rise to modifications of the GDH sum rule. In this letter, we estimate the contribution of these terms, and compare the result with that of the latest phenomenological analysis[5].

The crucial point of ref.[10] is that the vacuum expectation value of the triple commutator of charge densities generated by quark fields cannot vanish, and is proportional to the number of colors carried by these fields. Furthermore, its structure implies that the charge density commutator  $[V^a(\mathbf{x}), V^b(\mathbf{y})]$  must have a term symmetric in the  $a, b$  indices. This is in addition to the usual anti-symmetric term,  $if^{abc}V^c(\mathbf{x})\delta^3(\mathbf{x} - \mathbf{y})$ . Finally, the extra term must be of the form  $d^{abc}[\nabla \times \mathbf{b}_A^c(\mathbf{x})] \cdot \nabla \delta^3(\mathbf{x} - \mathbf{y})$  in order that the charge densities transform properly under global flavor transformations. Here,  $\mathbf{b}_A^c$  is an axial-vector operator whose precise form is dependent upon specific dynamics.

The minimal (infinite dimensional) Lie algebra that reproduces the correct vacuum expectation value of the triple commutators of the vector and axial-vector charge densities is given by the following relations[10]:

$$\begin{aligned}
[V^a(\mathbf{x}), V^b(\mathbf{y})] &= if^{abc}V^c(\mathbf{x})\delta^3(\mathbf{x} - \mathbf{y}) + \frac{i}{48\pi^2}d^{abc}[\nabla \times \mathbf{b}_A^c(\mathbf{x})] \cdot \nabla \delta^3(\mathbf{x} - \mathbf{y}), \\
[V^a(\mathbf{x}), A^b(\mathbf{y})] &= if^{abc}A^c(\mathbf{x})\delta^3(\mathbf{x} - \mathbf{y}) + \frac{i}{48\pi^2}d^{abc}[\nabla \times \mathbf{b}_V^c(\mathbf{x})] \cdot \nabla \delta^3(\mathbf{x} - \mathbf{y}), \\
[A^a(\mathbf{x}), A^b(\mathbf{y})] &= if^{abc}V^c(\mathbf{x})\delta^3(\mathbf{x} - \mathbf{y}) + \frac{i}{48\pi^2}d^{abc}[\nabla \times \mathbf{b}_A^c(\mathbf{x})] \cdot \nabla \delta^3(\mathbf{x} - \mathbf{y}), \\
[V^a(\mathbf{x}), \mathbf{b}_V^b(\mathbf{y})] &= if^{abc}\mathbf{b}_V^c(\mathbf{x})\delta^3(\mathbf{x} - \mathbf{y}) + 2iN_c\delta^{ab}\nabla\delta^3(\mathbf{x} - \mathbf{y}), \\
[A^a(\mathbf{x}), \mathbf{b}_A^b(\mathbf{y})] &= if^{abc}\mathbf{b}_V^c(\mathbf{x})\delta^3(\mathbf{x} - \mathbf{y}) + 2iN_c\delta^{ab}\nabla\delta^3(\mathbf{x} - \mathbf{y}), \\
[A^a(\mathbf{x}), \mathbf{b}_V^b(\mathbf{y})] &= if^{abc}\mathbf{b}_A^c(\mathbf{x})\delta^3(\mathbf{x} - \mathbf{y}), \\
[V^a(\mathbf{x}), \mathbf{b}_A^b(\mathbf{y})] &= if^{abc}\mathbf{b}_A^c(\mathbf{x})\delta^3(\mathbf{x} - \mathbf{y}).
\end{aligned} \tag{2}$$

The generators of the algebra are the charge densities, the  $\mathbf{b}$  terms and a central charge  $N_c$ , which is equal to the number of colors for the quarks. The number of colors did not appear in the older current algebras. Note that despite the appearance of an extension term in the charge density commutator, the electromagnetic current is not anomalous as a gauge current[11].

However, upon application of standard techniques[9, 12], the extension term does yield a modified form of the GDH sum rule:

$$\frac{2\pi^2\alpha}{M^2}(\kappa_{p,n})^2 + S_{p,n} = \int_{\omega_0}^{\infty} \frac{[\sigma_{3/2}(\omega) - \sigma_{1/2}(\omega)]_{p,n}}{\omega} d\omega. \quad (3)$$

Here the subscripts  $p, n$  label the proton and neutron sum rules. The additional contribution,  $S_{p,n}$ , is due to the symmetric term. The discrepancy found in the traditional sum rule, if it persists, is an indication that the term  $S_{p,n}$  does not vanish. In the following, we describe a scenario in which an  $S_{p,n}$  term appears naturally with a value that accounts for the apparent discrepancy in the sum rule.

We begin by observing that the operators  $\mathbf{b}_{V,A}$  defined by the charge density commutators in Eq.(2) share the same quantum numbers as the conventional vector and axial-vector currents. However, except in singular limits[10], they cannot be directly proportional as operators to these currents. The Schwinger terms in the commutators of the charge densities and the spatial current densities are infinite in QCD, while the corresponding quantity in Eq.(2) takes on the finite value  $2N_c$ .

Nevertheless, certain classes of matrix elements of  $\mathbf{b}_{V,A}$  may well be proportional to those of the currents. In the following, we will examine this possibility for the class of single-particle matrix elements of this operator. Specifically, we will assume that single-particle matrix elements of  $\mathbf{b}_{A,V}$  are well-approximated by contributions from the nucleon,  $\pi$ ,  $a_1$  and  $\rho$ , and that their relative strengths among these states are the same as the corresponding ones for the weak currents  $\mathbf{A}$  and  $\mathbf{V}$ . Such a hypothesis is tantamount to supposing that one may realize the extended algebra by local fields with these quantum numbers within a specific field-theoretic context. We symbolically write the relevant matrix elements as  $\mathbf{b}_A = K_A \mathbf{A}_L$ ,  $\mathbf{b}_V = K_V \mathbf{V}_L$ , where the subscript  $L$  indicates that only the couplings to the low-lying states are included.

Within such a framework, the  $\mathbf{b}$  operators are effectively decoupled from the high energy states, and two point functions of the form  $\langle b_A A \rangle$  and  $\langle b_V V \rangle$  are finite, consistent with Eq.(2). Assuming the validity of the KSRF relation[13], the spectral sum rule from  $\langle b_A A \rangle$  equals  $2f_\pi^2 K_A$ , half coming from the pion and the other half from  $a_1$ [14]. The spectral sum rule from  $\langle b_V V \rangle$ , in the same approximation, is given by  $2f_\pi^2 K_V$ . Comparing the spectral sums with the Schwinger terms in Eq.(2), we have, for  $N_c = 3$ ,

$$K_A = K_V = 3/f_\pi^2. \quad (4)$$

We may compare this result with what can be expected within the context of an

effective field theory for QCD at low energies, such as the non-linear sigma model[15, 16]. For simplicity, we restrict our attention to the minimal model involving only pions, and consider the 2-flavor case[17] (for a discussion of the case with more than two flavors, see [18]). Within this  $SU(2)$  framework there is no canonical isosinglet current, and the associated electromagnetic current cannot describe properties of the nucleon. As pointed out in[19], for purposes of studying these properties, one should include a topological current which is to be identified as the isosinglet baryon number current. The full electromagnetic current relevant for both the nucleon and the pion is the sum of the canonical isotriplet current and this topological current. Using canonical commutation relations, we can check that with this augmentation the algebra of Eq.(2) is indeed reproduced. More specifically, the commutator of the electromagnetic charge densities has an extension term that is purely isotriplet, and takes the form  $\mathbf{b}_A = K\mathbf{A}$ , where  $\mathbf{A}$  is the *exact* isotriplet axial-vector current in the model.

The precise value of  $K$  is dependent upon the dynamics relevant to the energy scale under consideration; it is given in the minimal non-linear sigma model by  $K_A = 24/f_\pi^2$ , which is much larger than the estimate quoted in Eq.(4). The difference is primarily due to the absence of the  $a_1$  and  $\rho$  mesons in this model. The KSFR relation assumed in our estimate above implies equal contributions of the pion and the  $a_1$  to

the Schwinger term coefficient of the extension current density in Eq.(2). Through the Weinberg sum rule, their sum gives the contribution of the  $\rho$  to the Schwinger term coefficient. Since this coefficient in Eq.(2) is fixed to be the number of colors  $N_c$ , the proportionality factor must be correspondingly larger to make up for the missing contributions from the spin-1 mesons.

We will be using Eq.(4) in the subsequent analysis. The contributions  $S_{p,n}$  to the extension term then have the values:

$$S_{(p,n)} = \alpha \frac{(g_A)_{(p,n)} K_A}{18} \quad (5)$$

$$(g_A)_{(p,n)} = \frac{1}{\sqrt{3}} G_A^{(8)} \pm G_A^{(3)} \quad (6)$$

when we substitute for the flavor  $SU(3)$   $d$ -symbols. The superscripts denote the directions in this flavor space, with  $G_A^{(3)} = 1.25$  being the Gamow-Teller  $\beta$ -decay constant. The value for  $G_A^{(8)}$  is not known directly at this time. However, if one supposes flavor  $SU(3)$  symmetry, then by fitting measured hyperon decay rates, one can roughly estimate that  $G_A^{(8)} = 0.65$ , based upon an  $F/D$  ratio of 0.63[20]. In this work, we ignore the contribution from the singlet axial current, since the EMC experiments[3] suggest that with the same  $F/D$  ratio,  $G_A^{(0)} \sim 0.12$ ,

We may eliminate all dependence on the poorly determined  $G_A^{(8)}$  by taking the

difference of the proton and neutron GDH sum rules:

$$\frac{2\pi^2\alpha}{M^2}[(\kappa_p)^2 - (\kappa_n)^2] + S = \int_{\omega_0}^{\infty} \frac{[\sigma_{3/2}(\omega) - \sigma_{1/2}(\omega)]_p - [\sigma_{3/2}(\omega) - \sigma_{1/2}(\omega)]_n}{\omega} d\omega. \quad (7)$$

This form is referred to as the isovector-isoscalar sum rule[4, 5]. The quantity  $S$  denotes the difference  $S_p - S_n$ . The first term on the left-hand-side is actually negative; with  $\kappa_p = 1.793$ , and  $\kappa_n = -1.913$ , it equals  $-28\mu\text{b}$ . The right-hand-side has been estimated in Refs.[4, 5] to lie between  $50\mu\text{b}$  and  $70\mu\text{b}$ . This range, however, clearly underestimates the uncertainty of this quantity, since no error has been assigned to contributions coming from the  $\gamma p \rightarrow \pi\pi N$  process. The integral has also been cut off at 2 GeV in the laboratory photon energy. Nevertheless, one can see a definite discrepancy in this component of the GDH sum rule.

The value of the extension term  $S$  obtained from Eqs.(4) and (5) is,

$$S = \frac{\alpha G_A^{(3)}}{3F_\pi^2}. \quad (8)$$

With  $\alpha = 1/137$ ,  $G_A^{(3)} = 1.25$ , and  $F_\pi = 93\text{MeV}$ , we obtain  $S = 137\mu\text{b}$ . Therefore the left-hand-side of Eq.(7) now has a total value  $-28\mu\text{b} + 137\mu\text{b} = 109\mu\text{b}$ , which brings Eq.(7) into qualitative agreement with the recent analyses of Ref.[5].

Using  $SU(3)$  flavor symmetry, we may also estimate the corrections to the individual proton and neutron GDH sum rules. The magnetic moment contribution to the left hand side of the proton sum rule is  $205\mu\text{b}$ . Using the result given in Eq.(4)

we have  $S_p = 89\mu\text{b}$ , which increases the total to  $294\mu\text{b}$ . The integral over the proton cross-sections is estimated to be  $260\mu\text{b}$ [5]. For neutrons, the magnetic moment contribution is  $233\mu\text{b}$ , while  $S_n = -48\mu\text{b}$ . The left-hand-side therefore equals  $185\mu\text{b}$ , which is to be compared with the estimate of  $190\mu\text{b}$  for the integral. Given the uncertainties involved in these comparisons, the modified GDH sum rules appear to be reasonably well-satisfied.

The above results involve real photons, with  $Q^2 = 0$ . What happens when  $Q^2 > 0$ , as in deep inelastic electron and muon scatterings? Recent analyses of the EMC[3] data suggest that the integrals over the moments of the polarized spin structure function, which are in effect the cross-sections appearing in GDH sum rules for virtual photons, must change sign as  $Q^2 \rightarrow 0$  if the original GDH sum rules without extension modifications are to be satisfied[2]. An exception had been the isovector-isoscalar integral, which is constrained to equal  $-G_A^{(3)}/6$  for large  $Q^2$  by the Bjorken sum rule[21]. This sum rule joins onto the isovector-isoscalar GDH sum rule, as  $Q^2 \rightarrow 0$ , and without the modification  $S$  in Eq.(7), one would have expected the integral to remain negative. No sign change appeared necessary for this integral. In actual fact, however, Eq.(7) and recent analyses[5] of available data at  $Q^2 = 0$  both suggest that there is a sign change in this case as well.

Why do these sign-flips occur? From our present perspective, such sign-flips rep-

resent the cross-over from short-distance physics, where perturbative QCD works, to large-distance non-perturbative physics. In perturbative QCD, the extension terms should vanish. We therefore do not expect any modifications of sum rules such as the Bjorken sum rule, which are valid for large values of  $Q^2$ . For smaller values of  $Q^2$ , however, we can approximate large-distance physics through contributions of low-lying resonances, and try to saturate the spectral sum rules with these states[14]. The success of the consequent predictions suggest that the **b** operator only has significant couplings to low-lying state, giving rise to finite values for the extension terms, and corrections to the GDH sum rules. As noted above, the relevant integrals change sign as a result.

The verification of any sum rule is an evolving process, since all comparisons must necessarily include cut-offs for the various integrals. Data from experiments planned at CEBAF, which will probe intermediate values of  $Q^2$ , can only shed more light on the physics behind the issues presented in this Letter.

This work was supported in part by the U.S. Department of Energy Grants DE-FG05-92ER40709, DE-FG05-88ER40454, and DE-FG02-91ER40685.

## References

- [1] S.B. Gerasimov, Yad. Fiz. **2** (1965) 598 [Sov. J. Nucl. Phys. **2** (1966) 430]; S.D. Drell and A.C. Hearn, Phys. Rev. Lett. **16** (1966) 908. For an earlier related work, see L. I. Lapidus and Chou Kuang-chao, JETP, **14** (1962) 357.
- [2] M. Anselmino, B.L. Ioffe and E. Leader, Yad. Fiz. **49** (1989) 214 [Sov. J. Nucl. Phys. **49** (1989) 136].
- [3] J. Ashman *et. al.*, Phys. Lett. **B 206** (1988) 364; Nucl.Phys. **B328** (1989) 1.
- [4] I. Karliner, Phys. Rev. **D 7** (1973) 2717.
- [5] R.L. Workman and R.A. Arndt, Phys. Rev.**D 45** (1992) 1789.
- [6] G.C. Fox and D.Z. Freedman, Phys. Rev. **182** (1969) 1628. See also S.B. Gerasimov, Yad. Fiz. **5** (1967) 1263 [Sov. J. Nucl. Phys. **5** (1967) 902].
- [7] H.D. Abarbanel and M.L. Goldberger, Phys. Rev. **165** (1968) 1594.
- [8] J. B. Bronzan, I. S. Gerstein, B. W. Lee and F. E. Low, Phys. Rev. Lett. **18** (1967) 32; Phys. Rev. **157** (1967) 1448.
- [9] K. Kawarabayashi and M. Suzuki, Phys. Rev. **150** (1966) 1362, *ibid*, **152** (1966) 1383; K. Kawarabayashi and W. Wada, *ibid*, **152** (1966) 1286.

- [10] L.N. Chang and Y. Liang, Phys. Lett. **B 268** (1991) 64; Phys. Rev. **D 45** (1992) 2121.
- [11] Current Algebra and Anomalies, eds. S. B. Treiman, R. Jackiw, B. Zumino, and E. Witten (Princeton University Press, Princeton, 1985).
- [12] S. Adler and R. Dashen, Current Algebra (Benjamin, New York, 1968).
- [13] K. Kawarabayashi and Suzuki, Phys. Rev. Lett. **16** (1966) 255; Riazuddin and Fayyazuddin, Phys. Rev. **147** (1966) 1071.
- [14] S. Weinberg, Phys. Rev. Lett., **18** (1967) 507.
- [15] S. Weinberg, Phys. Rev., **166**, (1968) 1568.
- [16] J. Wess and B. Zumino, Phys. Lett., **B 37** (1971) 95.
- [17] Y. Liang, unpublished.
- [18] G. Kramer and K. Meetz, Z. Phys. **C 30** (1986) 317.
- [19] E. Witten, Nucl. Phys. **B223** (1983) 433.
- [20] M. Bourquin *et.al*, Z. Phys. **C21** (1983) 27.
- [21] J. D. Bjorken, Phys. Rev. **148** (1966) 1467.